


# Multiple Linear Regression: John McGready



**Section D**

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Handling Multiple Categorical Predictors in Multiple Linear Regression: ANOVA as a Regression Model

**The Situation**

- Sometimes regression scenarios include predictors that are not continuous, not binary, but multi-categorical
- Examples
  - Subject's race (White, African-American, Hispanic, Asian, Other)
  - City of residence (Baltimore, Chicago, Tokyo, Madrid)

2

**The Situation**

- How can this type of situation be handled in a regression framework?
- We'll explore with an example using a data set containing information about average SAT scores in 51 U.S. states (treating D.C. as a state)—the averages were based on random samples of students taken within each of the 51 states

3

**SAT Scores Example**

- The SAT (Scholastic Aptitude Test) is taken by many U.S. high school students to fulfill requirements for entry into most colleges or universities
- The test is made up of two components: verbal and quantitative (math)
- This analysis will use the quantitative score, which ranges from 200-800 (we will refer to these simply as SAT scores for simplicity)
- This data comes from the book *Statistics with Stata 8*, by Lawrence Hamilton

4

**SAT Scores Example**

- Data consists of 51 observations: the cumulative average SAT quantitative section scores for the 51 U.S. states for students taking the test in 1990
- Additional information on each observation includes geographical region of the state (West, Northeast, South, Midwest) and per-pupil education expenditures in each state in 1990

5

**SAT Scores Example**

- A key question
  - Do average SAT scores differ across the four regions of the country and, if so, what is the magnitude of these differences?

6

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**Snippet of the Data**

- Here is a snippet of the data in Stata (msat is mean math SAT score for the state, region is a "labeled" numerical variable)

```

+-----+
| msat   region |
+-----+
1. | 515     South |
2. | 481     West  |
3. | 490     West  |
4. | 523     South |
5. | 482     West  |
+-----+
6. | 506     West  |
7. | 468     N. East|

```

7

**ANOVA**

- Analysis of various testing for differences between the four states yields a p-value of less than 0.001
- So there are at least some statistical differences in SAT quantitative scores across the four regions—but in order to find out which regions are statistically different and to figure out by how large (and in what direction) these differences occur would require a lot of t-tests

8

**ANOVA as a Regression Model**

- Could this analysis be done by a linear regression relating SAT scores to region?
- How can we handle a predictor that takes on four categories?

9

**ANOVA as a Regression Model**

- Arbitrarily give each region a numerical value ( $x_i = 1$  for Western region states, 2 for Northeastern states, 3 for Southern states, and 4 for mid-Western states for example) and fit SLR of

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

- Where  $\hat{y}$  is estimated mean SAT score, and  $x_i$  is region as defined above

10

**ANOVA as a Regression Model**

- This is not a good idea!!!
- Coding is arbitrary, could have assigned  $x_i = 1$  for Midwest, etc. . . .
  - Estimated coefficient of region will depend on arbitrary coding
- Coding "assumes" mean SAT score differences between regions is "incremental"
  - Example—difference in average SAT scores between Southern states ( $x_i = 3$ ) and Western States ( $x_i = 1$ ) is twice the difference between Northeastern States ( $x_i = 2$ ) and Western States ( $x_i = 1$ )

11

**ANOVA as a Regression Model**

- Alternative approach—designate one region as "reference" region, say Western region, and make binary indicators for each of the three other regions
  - $x_1 = 1$  if Northeastern state, 0 otherwise
  - $x_2 = 1$  if Southern state, 0 otherwise
  - $x_3 = 1$  if mid-Western state, 0 otherwise

12

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**ANOVA as a Regression Model**

- Here is a table showing the x values for each region

Region	$x_1$	$x_2$	$x_3$
West	0	0	0
Northeast	1	0	0
South	0	1	0
Mid-west	0	0	1

13

**ANOVA as a Regression Model**

- Fit the regression model

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3$$

- Here, each coefficient estimates mean SAT score difference between a region that has a corresponding x value of 1 and the reference region (Western states)
- Notice, the intercept has meaning here—it's the estimated mean when all xs are 0, the estimated mean SAT score for Western states!

14

**ANOVA as a Regression Model**

- Example
  - For Northeastern states ( $x_1 = 1, x_2 = 0, x_3 = 0$ ) model predicts
 
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 * 1 + \hat{\beta}_2 * 0 + \hat{\beta}_3 * 0$$

$$= \hat{\beta}_0 + \hat{\beta}_1$$
  - For Western states ( $x_1 = 0, x_2 = 0, x_3 = 0$ ) model predicts
 
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 * 0 + \hat{\beta}_2 * 0 + \hat{\beta}_3 * 0$$

$$= \hat{\beta}_0$$

15

**ANOVA as a Regression Model**

- Example
  - So  $\hat{y}_{NE} - \hat{y}_W = \hat{\beta}_0 + \hat{\beta}_1 - \hat{\beta}_0 = \hat{\beta}_1$
- Similar results can be shown for other coefficients

16

**ANOVA as a Regression Model**

- Stata results
- Notice, data in the following format . . .

```

+-----+
msat  region
-----+-----
1.   515      3
2.   481      1
3.   490      1
4.   523      3
5.   482      1
-----+-----
6.   506      1
7.   468      2
8.   464      3
    
```

17

**ANOVA as a Regression Model**

- "xi" option before regression command will automatically create binary indicators for a multi-categorical variable
- Syntax
  - xi: regress msat i.region

18

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**ANOVA as a Regression Model**

- Stata results

```

. xli regress mmat i.region
i.region      _iregion_1-4      (naturally coded; _iregion_1 omitted)

-----+-----
Source |      SS      df      MS              Number of obs =   50
-----+-----
Model | 26433.6728    3 8911.22428          F( 3, 46) = 12.26
Residual | 33050.8072   46 718.495808          Prob > F      = 0.0000
Total | 59484.48     49 1213.96898          R-squared     = 0.4444
                                           Adj R-squared = 0.4081
                                           Root MSE    = 26.805

-----+-----
mmat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
_iregion_2 | -32.01709   11.62333    -2.75  0.008   -55.41364   -8.620547
_iregion_3 | -11.64904   10.00874    -1.16  0.250   -31.79559    8.497512
_iregion_4 | 35.45513    10.7305    3.30  0.002   13.85576    57.0545
     _cons | 498.4615   7.434306    67.05  0.000   483.4971   513.426
    
```

19

**Stata Results**

- Resulting regression equation

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3$$

$$\hat{y} = 498.5 - 32.0x_1 - 11.6x_2 + 35.5x_3$$

20

**ANOVA as a Regression Model**

- Overall F-test

```

. xli regress mmat i.region
i.region      _iregion_1-4      (naturally coded; _iregion_1 omitted)

-----+-----
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```

21

**ANOVA as a Regression Model**

- This is the overall test for . . .
  - $H_0: \beta_1 = \beta_2 = \beta_3$ ; no differences in mean SAT scores across the four regions
  - $H_a$ : at least one region has different mean SAT scores than the others
- This is the same exact test that we did with the traditional ANOVA approach

22

**ANOVA as a Regression Model**

- Some of the estimated regional differences

```

. xli regress mmat i.region
i.region      _iregion_1-4      (naturally coded; _iregion_1 omitted)

-----+-----
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     _cons | 498.4615   7.434306    67.05  0.000   483.4971   513.426
    
```

23

**Results**

- A statistically significant relationship was found between mean SAT scores and student's region of the country ( $p < .0001$  by F-test)
- Students from northeastern states had SAT scores of 32 points lower on average than students from western states (95% CI 8.6 to 55.4 points lower)

24

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**Results**

- Students from southern states had SAT scores of 11.6 points lower on average than students from western states (95% CI 31.6 points lower to 8.5 points higher)
- Students from mid-western states had SAT scores of 35.5 points higher on average than students from western states (95% CI 13.9 points to 57.0 points higher)
- Regional differences account for 44% of the variation in SAT scores

25

**Results**

- What about other comparisons—for example, SAT scores for Northeastern states to mid-western states?
  - One approach—recode indicators for region making “mid-west” the reference group—more work!
  - Another option—use existing coefficients

26

**Results**

- Recall  $\hat{\beta}_1 = \hat{y}_{NE} - \hat{y}_W = -32.0$  estimates the average difference in SAT scores for northeastern states minus (compared to) western states
- Recall  $\hat{\beta}_3 = \hat{y}_{MW} - \hat{y}_W = 35.5$  estimates the average difference in SAT scores for mid-western states minus (compared to) western states

27

**Results**

- So :
 
$$\hat{\beta}_1 - \hat{\beta}_3 = \hat{y}_{NE} - \hat{y}_W - (\hat{y}_{MW} - \hat{y}_W)$$

$$= \hat{y}_{NE} - \hat{y}_W - \hat{y}_{MW} + \hat{y}_W$$

$$= \hat{y}_{NE} - \hat{y}_{MW}$$
- So the estimated mean difference in SAT scores between northeastern states and mid-western states is given by (-32.0-35.5) = -67.5 points

28

**Results**

- We can employ Stata to do this and get a 95% CI (just FYI)
- The “lincom” command can be run after any regression to give estimates for differences in coefficients

29

**ANOVA as a Regression Model**

- We need to use names Stata gives to coefficients in command

```

    . x1: regress mmat i.region
    i.region      _i.region_1-4      (naturally coded; _i.region_1 omitted)

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    Root MSE   = 26.805

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    -----+-----
    _i.region_2 | -32.01709   11.62333     -2.75  0.008   -55.41364   -8.620547
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    _cons | 498.4615    7.434306    67.05  0.000   483.4971   513.426
    
```

30

